Third Grade - Mathematics

Kentucky Core Academic Standards with Targets

Student Friendly Targets

Pacing Guide

Anderson County Schools

Safe, Prepared, Responsible

College and Career Readiness Anchor Standards for Math

The K-5 standards on the following pages define what students should understand and be able to do by the end of each grade. They correspond to eight mathematical practices: 1) Make sense of problems and persevere in solving them, 2) Reason abstractly and quantitatively, 3) Construct viable arguments and critique the reasoning of others, 4) Model with mathematics, 5) Use appropriate tools strategically, 6) Attend to precision, 7) Look for and make use of structure, and 8) Look for express regularity in repeated reasoning.

Mathematics is divided into five domains: 1) Counting and Cardinality (CC), 2) Operations and Algebraic Thinking (OA), 3) Number and Operations in Base Ten (NBAT), 4) Measurement and Data (MD), and 5) Geometry (G).

Development of Pacing Document

During the summer 2011, Anderson County teachers and administrators developed learning targets for each of the Kentucky Core Content Standards. In winter 2012, curriculum resource teachers verified the congruency of the standards and targets and recommended revisions. Teachers refined the work and began planning the development of common assessments to ensure students learn the intended curriculum. Anderson County Schools would like to thank each of our outstanding teachers and administrators who contributed to this important math curriculum project. Special thanks to Robin Arnzen, Stephanie Barnes, Traci Beasley, Julie Bowen, Tony Calvert, Linda Dadisman, Amanda Ellis, Leslie Fields, Amy Gritton, Lauren Hamel, Linda Hill, Sharon Jackman, Lesley Johnson, Steve Karsner, Chris Kidwell, Joel Maude, Melissa Montgomery, Matt Ogden, Kim Penn, Wayne Reese, Monica Rice, Chrystal Rowland, Kim Ruble, Jennifer Sallee, Amy Satterly, Krista Sawyer, Francine Sloan, Jeanna Slusher, Shayla Smith, T.J. Spivey, Rebecca Stevens, Emily Thacker, Lori Wells, Shannon Wells, Tim Wells, and Jamie White. Thanks also to Tony Calvert (EBW), Brian Edwards (ACHS), and Alex Hunter (ACMS) for providing comments to the work.

North Carolina State Board of Education created a most helpful document entitled “Common Core Instructional Support Tools - Unpacking Standards”. The document answers the question “What do the standards mean that a student must know and be able to do?” The “unpacking” is included in our “What Does This Standard Mean?” section. The complete North Carolina document can be found at http://www.dpi.state.nc.us/docs/acre/standards/common-core-tools/unpacking/math/3rd.pdf

Coding for Math Document

Domain

Standard Number

Grade Level

KY Core Academic Standard (KCAS) Bold, 14 pt

3.OA.7 fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 x 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers

Target Type

Knowledge Targets:

☐ Know from memory all products of two one-digit numbers.

I can define "round or rounding in relation to place value.

KDE Target Regular, 12 pt

Reasoning Targets:

Level Standard is Assessed Highlighted

☐ Analyze a multiplication or division problem in order to choose an appropriate strategy to fluently multiply or divide within 10.

I can analyze a multiplication or division problem so that can choose a strategy to help me fluently multiply or divide within 10.

AC Target *Bold, Italics, 12 pt*

Anderson County Elementary
Pacing Guide

Math
Grade 3

Number Operations in Base Ten		
Standard	**What Does This Standard Mean?**	**Dates Taught**
3.NBT1: Use place value understanding to round whole numbers to the nearest 10 or 100. **Knowledge Targets:** ☐ Define “round or rounding” in relation to place value. (Underpinning target) ***I can define “round or rounding” in relation to place value.*** *(Underpinning target).* ☐ Round a whole number to the nearest 10. ***I can round a whole number to the nearest 10.*** ☐ Round a whole number to the nearest 100. ***I can round a whole number to the nearest 100.***	This standard refers to place value understanding, which extends beyond an algorithm or procedure for rounding. The expectation is that students have a deep understanding of place value and number sense and can explain and reason about the answers they get when they round. Students should have numerous experiences using a number line and a hundreds chart as tools to support their work with rounding.	August
Addition/Subtraction Problem Solving		
3.NBT.2 Fluently add and subtract within 1,000 using strategies and algorithms based on place value, properties of operations, and or the relationship between addition and subtraction. **Knowledge Targets:** ☐ Know strategies and algorithms for adding and subtracting within 1000.	This standard refers to fluently, which means accuracy, efficiency (using a reasonable amount of steps and time), and flexibility (using strategies such as the distributive property). The word algorithm refers to a procedure or a series of steps. There are other algorithms other than the standard algorithm. Third grade students should have experiences beyond the standard algorithm. A variety of algorithms will be assessed on EOG. Problems should include both vertical and horizontal forms, including opportunities for students to apply the commutative and associative properties. Students explain their thinking and show their work by using strategies and algorithms, and verify that their answer is reasonable.	September

I can use strategies and algorithms for adding and subtracting within 1000.

☐ Fluently add and subtract within 1000.

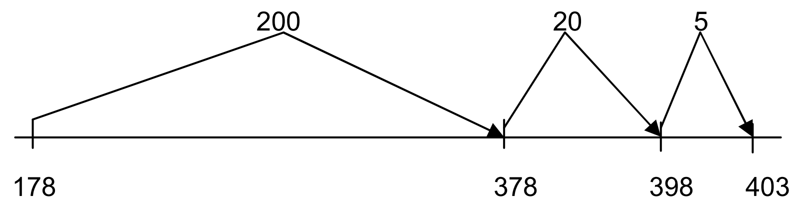
I can fluently add and subtract within 1000.

Example:
There are 178 fourth graders and 225 fifth graders on the playground. What is the total number of students on the playground?

Student 1		Student 2		Student 3
100 + 200 = 300 70 + 20 = 90 8 + 5 = 13 300 + 90 + 13 = 400 students		I added 2 to 178 to get 180. I added 220 to get 400. I added the 3 left over to get 403.		I know the 75 plus 25 equals 100. I then added 1 hundred from 178 and 2 hundreds from 275. I had a total of 4 hundreds and I had 3 more left to add. So I have 4 hundreds plus 3 more which is 403.

Student 4

178 + 225 = ?
178 + 200 = 378
378 + 20 = 398
398 + 5 = 403



3.OA.8 Solve two step word problems using the four operations. Represent these problems using equations with the letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Knowledge Targets:

- ☐ Know the order of operations (without parentheses).

I can explain the order of operations (without parentheses).

- ☐ Know strategies for estimating.

I can use strategies for estimating.

Reasoning Targets:

- ☐ Construct an equation with a letter standing for the unknown quantity.

I can construct an equation with a letter standing for the unknown quantity.

- ☐ Solve two-step word problems using the four operations.

I can solve two-step word problems using the four operations.

- ☐ Justify your answer using various estimation strategies.

I can justify my answer using various estimation strategies.

This standard refers to two-step word problems using the four operations. The size of the numbers should be limited to related 3rd grade standards (e.g., 3.OA.7 and 3.NBT.2). Adding and subtracting numbers should include numbers within 1,000, and multiplying and dividing numbers should include single-digit factors and products less than 100.

This standard calls for students to represent problems using equations with a letter to represent unknown quantities.

Example:
Mike runs 2 miles a day. His goal is to run 25 miles. After 5 days, how many miles does Mike have left to run in order to meet his goal? Write an equation and find the solution ($2 \times 5 + m = 25$).

This standard refers to estimation strategies, including using compatible numbers (numbers that sum to 10, 50, or 100) or rounding. The focus in this standard is to have students use and discuss various strategies. Students should estimate during problem solving, and then revisit their estimate to check for reasonableness.

Example:
Here are some typical estimation strategies for the problem:
On a vacation, your family travels 267 miles on the first day, 194 miles on the second day and 34 miles on the third day. How many total miles did they travel?

Student 1 I first thought about 267 and 34. I noticed that their sum is about 300. Then I knew that 194 is close to 200. When I put 300 and 200 together, I get 500.
Student 2 I first thought about 194. It is really close to 200. I also have 2 hundreds in 267. That gives me a total of 4 hundreds. Then I have 67 in 267 and the 34. When I put 67 and 34 together that is really close to 100. When I add that hundred to the 4 hundreds that I already had. I end up with 500.

October

	Student 3 I rounded 267 to 300. I rounded 194 to 200. I rounded 34 to 30. When I added 300, 200 and 30. I know my answer will be about 530. The assessment of estimation strategies should only have one reasonable answer (500 or 530), or a range (between 500 and 550). Problems should be structured so that all acceptable estimation strategies will arrive at a reasonable answer.	
3.OA.9: Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. **Knowledge Targets:** ☐ Identify arithmetic patterns (such as even and odd numbers, patterns an an addition table, patterns in a multiplication table, patterns regarding multiples and sums.) ***I can identify arithmetic patterns(such as even and odd numbers, patterns in an addition table, patterns in a multiplication table, patterns regarding multiples and sums)*** **Reasoning Targets:** ☐ Explain rules for a pattern using properties of operations. (Properties of operations, glossary page 90 Common Core State Standards.) ***I can explain rules for a pattern using properties of operations.*** ☐ Explain relationships between the numbers in a pattern.	This standard calls for students to examine arithmetic patterns involving both addition and multiplication. Arithmetic patterns are patterns that change by the same rate, such as adding the same number. For example, the series 2, 4, 6, 8, 10 is an arithmetic pattern that increases by 2 between each term. This standards also mentions identifying patterns related to the properties of operations. Examples: • Even numbers are always divisible by 2. Even numbers can always be decomposed into 2 equal addends (14 = 7 + 7). • Multiples of even numbers (2, 4, 6, and 8) are always even numbers. • On a multiplication chart, the products in each row and column increase by the same amount (skip counting). • On an addition chart, the sums in each row and column increase by the same amount. What do you notice about the numbers highlighted in pink in the multiplication table? Explain a pattern using properties of operations. *When (commutative property) one changes the order of the factors they will still gets the same product, example 6 x 5 = 30 and 5 x 6 = 30.*	

I can explain relationships between numbers in a pattern.

x	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

Teacher: What pattern do you notice when 2, 4, 6, 8, or 10 are multiplied by any number (even or odd)?

Student: The product will always be an even number.
Teacher: Why?

x	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

What patterns do you notice in this addition table? Explain why the pattern works this way?

x	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

Students need ample opportunities to observe and identify important numerical patterns related to operations. They should build on their previous experiences with properties related to addition and subtraction. Students investigate addition and multiplication tables in search of patterns and explain why these patterns make sense mathematically.

Example:

- Any sum of two even numbers is even.
- Any sum of two odd numbers is even.
- Any sum of an even number and an odd number is odd.
- The multiples of 4, 6, 8, and 10 are all even because they can all be decomposed into two equal groups.
- The doubles (2 addends the same) in an addition table fall on a diagonal while the doubles (multiples of 2) in a multiplication table fall on horizontal and vertical lines.
- The multiples of any number fall on a horizontal and a vertical line due to the commutative property.
- All the multiples of 5 end in a 0 or 5 while all the multiples of 10 end with 0. Every other multiple of 5 is a multiple of 10.

Students also investigate a hundreds chart in search of addition and subtraction patterns. They record and organize all the different possible sums of a number and explain why the pattern makes sense.

addend	addend	sum
0	20	20
1	19	20
2	18	20
3	17	20
4	16	20
☐	☐	☐
☐	☐	☐
☐	☐	☐
20	0	20

Multiplication

3.OA.1 Interpret products of whole numbers, e.g., Interpret 5x7 as the total number of objects in five groups of seven objects each.

Knowledge Targets:

- ☐ Find the product of multiple groups of objects.

I can find the product of multiple groups of objects.

Reasoning Targets:

- ☐ Interpret products of whole numbers as a total number of objects in a number of groups.

I can interpret products of whole numbers as a total number of objects in a number of groups.

This standard interpret products of whole numbers. Students recognize multiplication as a means to determine the total number of objects when there are a specific number of groups with the same number of objects in each group. Multiplication requires students to think in terms of groups of things rather than individual things. Students learn that the multiplication symbol 'x' means "groups of" and problems such as 5 x 7 refer to 5 groups of 7.

Example:
Jim purchased 5 packages of muffins. Each package contained 3 muffins. How many muffins did Jim purchase?
5 groups of 3, 5 x 3 = 15. Describe another situation where there would be 5 groups of 3 or 5 x 3.

November

3.OA.3: Use multiplication and division within

This standard references various strategies that can be used to solve word problems involving multiplication and division. Students should

100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

Knowledge Targets:

- ☐ Multiply and divide within 100.

I can multiply and divide within 100.

Reasoning Targets:

- ☐ Solve word problems in situations involving equal groups, arrays, and measurement quantities.

I can solve word problems in situations involving equal groups, arrays, and measurement quantities.

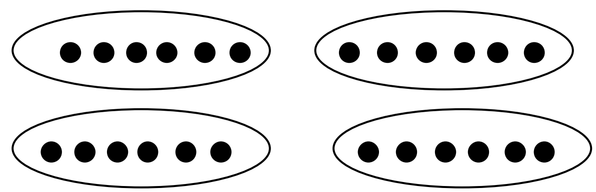
- ☐ Represent a word problem using a picture, an equation with a symbol for the unknown number, or in other ways.

I can show a word problem using a picture, an equation with a symbol for the unknown number, or in other ways.

apply their skills to solve word problems. Students should use a variety of representations for creating and solving one-step word problems, such as: If you divide 4 packs of 9 brownies among 6 people, how many cookies does each person receive? *(4 x 9 = 36, 36 ÷ 6 = 6).* Glossary page 89, Table 2 (table also included at the end of this document for your convenience) gives examples of a variety of problem solving contexts, in which students need to find the product, the group size, or the number of groups. Students should be given ample experiences to explore all of the different problem structures.

Examples of multiplication:
There are 24 desks in the classroom. If the teacher puts 6 desks in each row, how many rows are there?
This task can be solved by drawing an array by putting 6 desks in each row.
This is an array model

This task can also be solved by drawing pictures of equal groups.
4 groups of 6 equals 24 objects



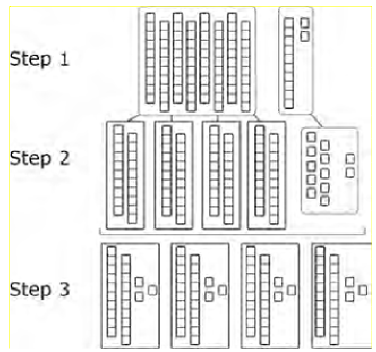
A student could also reason through the problem mentally or verbally, "I know 6 and 6 are 12. 12 and 12 are 24. Therefore, there are 4 groups of 6 giving a total of 24 desks in the classroom."

A number line could also be used to show equal jumps.

Students in third grade students should use a variety of pictures, such as stars, boxes, flowers to represent unknown numbers (variables).. Letters are also introduced to represent unknowns in third grade.

Examples of Division:
There are some students at recess. The teacher divides the class into 4 lines with 6 students in each line. Write a division equation for this story and determine how many students are in the class *(□ ÷ 4 = 6. There are 24students in the class).*

Determining the number of objects in each share (partitive division, where the size of the groups is unknown):
Example: The bag has 92 hair clips, and Laura and her three friends want to share them equally. How many hair clips will each person receive?



Determining the number of shares (measurement division, where the number of groups is unknown)

Example:
Max the monkey loves bananas. Molly, his trainer, has 24 bananas. If she gives Max 4 bananas each day, how many days will the bananas last?

Starting	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6
24	24–4=20	20-4=16	16-4=12	12- 4 = 8	8 – 4= 4	4 -4 = 0

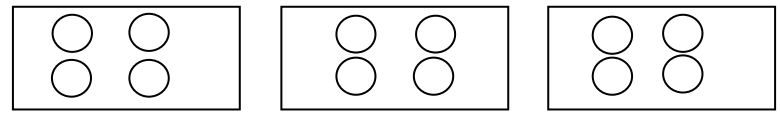
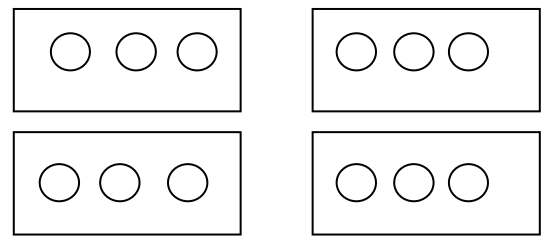
Solution: The bananas will last for 6 days.

3.OA.4: Determine the unknown whole number in a multiplication or division equation relating three whole numbers.

This standard refers to Glossary page 89, Table 2 (table also included at the end of this document for your convenience) and equations for the different types of multiplication and division problem structures. The easiest problem structure includes Unknown Product (3 x 6 = ? or 18 ÷ 3 =

Knowledge Targets: ☐ Multiply and divide within 100. ***I can multiply and divide within 100. (Underpinning)*** **Reasoning Targets:** ☐ Determine which operation (multiplication or division) is needed to determine the unknown whole number. ***I can choose which operation (multiplication or division) is needed to solve for the unknown. (Unknown means, the missing value.)*** ☐ Solve to find the unknown whole number in a multiplication or division equation. ***I can solve to find the unknown in a multiplication or division equation.***	6). The more difficult problem structures include Group Size Unknown (3 x ? = 18 or 18 ÷ 3 = 6) or Number of Groups Unknown (? x 6 = 18, 18 ÷ 6 = 3). The focus of 3.OA.4 goes beyond the traditional notion of *fact families*, by having students explore the inverse relationship of multiplication and division. Students apply their understanding of the meaning of the equal sign as "the same as" to interpret an equation with an unknown. When given 4 x ? = 40, they might think: • 4 groups of some number is the same as 40 • 4 times some number is the same as 40 • I know that 4 groups of 10 is 40 so the unknown number is 10 • The missing factor is 10 because 4 times 10 equals 40. Equations in the form of a x b = c and c = a x b should be used interchangeably, with the unknown in different positions. Example: Solve the equations below: 24 = ? x 6 72 ÷ = 9 Rachel has 3 bags. There are 4 marbles in each bag. How many marbles does Rachel have altogether? 3 x 4 = *m*	
3.OA.7: Fluently multiply and divide within the 100 using strategies such as the relationship between multiplication and division. (Example: Knowing that 8x5=40, one knows 40/5=8) or properties of operations. By the end of grade three, know from memory all products of two one digit numbers. **Knowledge Targets:** ☐ Know from memory all products of two one-digit numbers. ***I can identify all products of two one-digit numbers from memory.***	This standard uses the word fluently, which means accuracy, efficiency (using a reasonable amount of steps and time), and flexibility (using strategies such as the distributive property). "Know from memory" should not focus only on timed tests and repetitive practice, but ample experiences working with manipulatives, pictures, arrays, word problems, and numbers to internalize the basic facts (up to 9 x 9). By studying patterns and relationships in multiplication facts and relating multiplication and division, students build a foundation for fluency with multiplication and division facts. Students demonstrate fluency with multiplication facts through 10 and the related division facts. Multiplying and dividing fluently refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently. Strategies students may use to attain fluency include: • Multiplication by zeros and ones • Doubles (2s facts), Doubling twice (4s), Doubling three times (8s)	

Reasoning Targets: ☐ Analyze a multiplication or division problem in order to choose an appropriate strategy to fluently multiply or divide within 100. ***I can analyze a multiplication or division problem in order to choose an appropriate strategy to fluently multiply or divide within 100.***	• Tens facts (relating to place value, 5 x 10 is 5 tens or 50) • Five facts (half of tens) • Skip counting (counting groups of __ and knowing how many groups have been counted) • Square numbers (ex: 3 x 3) • Nines (10 groups less one group, e.g., 9 x 3 is 10 groups of 3 minus one group of 3) • Decomposing into known facts (6 x 7 is 6 x 6 plus one more group of 6) • Turn-around facts (Commutative Property) • Fact families (Ex: 6 x 4 = 24; 24 ÷ 6 = 4; 24 ÷ 4 = 6; 4 x 6 = 24) • Missing factors General Note: Students should have exposure to multiplication and division problems presented in both vertical and horizontal forms.	
3.NBT.3: Multiply one digit whole numbers by multiples of 10, in the range 10-90 (e.g. 9 x 80, 5 x 60) using strategies based on place value and properties of operations. **Knowledge Targets:** ☐ Know strategies to multiply one-digit numbers by multiples of 10 (up to 90). ***I can use strategies to multiply one-digit numbers by multiples of 10 (up to 90).*** **Reasoning Targets:** ☐ Apply knowledge of place value to multiply one-digit whole numbers by multiples of 10 (up to 90). ***I can use knowledge of place value to multiply one-digit numbers by multiples of 10 up to 90. That means, 2 x 10; 2 x 20; 2 x 30; 2 x 40, etc.***	This standard extends students' work in multiplication by having them apply their understanding of place value. This standard expects that students go beyond tricks that hinder understanding such as "just adding zeros" and explain and reason about their products. For example, for the problem 50 x 4, students should think of this as 4 groups of 5 tens or 20 tens. Twenty tens equals 200.	
Division		
3.OA.2: Interpret whole-number quotients of whole numbers, e.g. interpret 56/8 as the	This standard focuses on two distinct models of division: partition models and measurement (repeated subtraction) models.	December

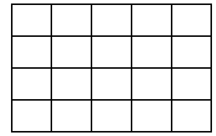
number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. **Knowledge Targets:** ☐ Know what the numbers in a division problem represent. ***I can identify what the numbers in a division problem represent.*** **Reasoning Targets:** ☐ Explain what division means and how it relates to equal shares. ***I can explain what division means and how it relates to partitioned equal shares.*** ☐ Interpret quotients as the number of shares or the number of groups when a set of objects is divided equally. ***I can interpret quotients as a number of shares or a number of groups when a set of objects is divided equally.***	Partition models focus on the question, "How many in each group?" A context for partition models would be: There are 12 cookies on the counter. If you are sharing the cookies equally among three bags, how many cookies will go in each bag?  Measurement (repeated subtraction) models focus on the question, "How many groups can you make?" A context for measurement models would be: There are 12 cookies on the counter. If you put 3 cookies in each bag, how many bags will you fill? 	
3.OA.4: Determine the unknown whole number in a multiplication or division equation relating three whole numbers. **Knowledge Targets:** ☐ Multiply and divide within 100. ***I can multiply and divide within 100. (Underpinning)*** **Reasoning Targets:** ☐ Determine which operation (multiplication or	This standard refers to Glossary page 89, Table 2 (table also included at the end of this document for your convenience) and equations for the different types of multiplication and division problem structures. The easiest problem structure includes Unknown Product (3 x 6 = ? or 18 ÷ 3 = 6). The more difficult problem structures include Group Size Unknown (3 x ? = 18 or 18 ÷ 3 = 6) or Number of Groups Unknown (? x 6 = 18, 18 ÷ 6 = 3). The focus of 3.OA.4 goes beyond the traditional notion of *fact families*, by having students explore the inverse relationship of multiplication and division. Students apply their understanding of the meaning of the equal sign as "the same as" to interpret an equation with	

division) is needed to determine the unknown whole number. ***I can choose which operation (multiplication or division) is needed to solve for the unknown. (Unknown means, the missing value.)*** ☐ Solve to find the unknown whole number in a multiplication or division equation. ***I can solve to find the unknown in a multiplication or division equation.***	an unknown. When given 4 x ? = 40, they might think: • 4 groups of some number is the same as 40 • 4 times some number is the same as 40 • I know that 4 groups of 10 is 40 so the unknown number is 10 • The missing factor is 10 because 4 times 10 equals 40. Equations in the form of a x b = c and c = a x b should be used interchangeably, with the unknown in different positions. Example: Solve the equations below: 24 = ? x 6 72 ÷ = 9 Rachel has 3 bags. There are 4 marbles in each bag. How many marbles does Rachel have altogether? 3 x 4 = *m*	
Problem Solving with Multiplication and Division **Geometry**		
3.OA.5: Apply properties of operations as strategies to multiply and divide. **Knowledge Targets:** ☐ Multiply and divide within 100. ***I can multiply and divide within 100.*** **Reasoning Targets:** ☐ Explain how the properties of operations work. ***I can explain how the properties of operations work.*** ☐ Apply properties of operations as strategies to multiply and divide. ***I can use properties of operations as strategies to multiply and divide.***	This standard references properties (rules about hoe numbers work) of multiplication. While students DO NOT need to not use the formal terms of these properties, student should understand that properties are rules about how numbers work, they need to be flexibly and fluently applying each of them. Students represent expressions using various objects, pictures, words and symbols in order to develop their understanding of properties. They multiply by 1 and 0 and divide by 1. They change the order of numbers to determine that the order of numbers does not make a difference in multiplication (but does make a difference in division). Given three factors, they investigate changing the order of how they multiply the numbers to determine that changing the order does not change the product. They also decompose numbers to build fluency with multiplication. The associative property states that the sum or product stays the same when the grouping of addends or factors is changed. For example, when a student multiplies 7 x 5 x 2, a student could rearrange the numbers to first multiply 5 x 2 = 10 and then multiply 10 x 7 = 70. The commutative property (order property) states that the order of numbers does not matter when you are adding or multiplying numbers. For example, if a student knows that 5 x 4 =	January

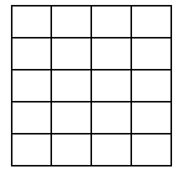
20, then they also know that 4 x 5 = 20.
The array below could be described as a 5 x 4 array for 5 columns and 4 rows, or a 4 x 5 array for 4 rows and 5 columns. There is no “fixed” way to write the dimensions of an array as rows x columns or columns x rows.

Students should have flexibility in being able to describe both dimensions of an array.

Example:



4 x 5 or 5 x 4

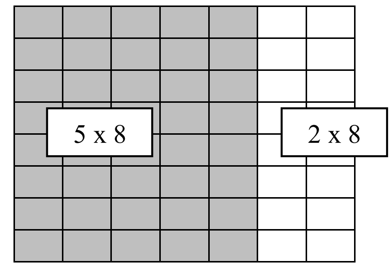


4 x 5 or 5 x 4

Students are introduced to the distributive property of multiplication over addition as a strategy for using products they know to solve products they don’t know. Students would be using mental math to determine a product. Here are ways that students could use the distributive property to determine the product of 7 x 6. Again, students **should use the distributive property, but can refer to this in informal language such as “breaking numbers apart”.**

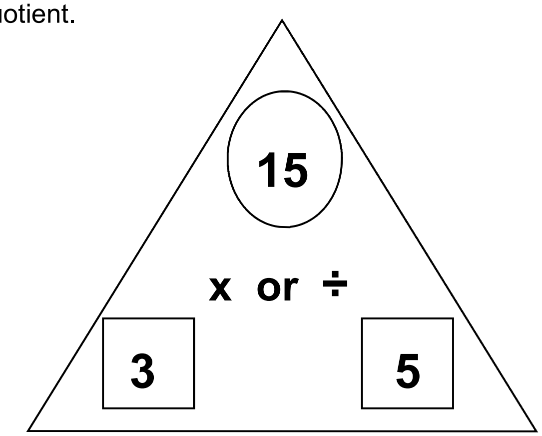
Student 1	Student 2	Student 3
7 x 6	7 x 6	7 x 6
7 x 5 = 35	7 x 3 = 21	5 x 6 = 30
7 x 1 = 7	7 x 3 = 21	2 x 6 = 12
35 + 7 = 42	21 + 21 = 42	30 + 12 = 42

Another example if the distributive property helps students determine the products and factors of problems by breaking numbers apart. For example, for the problem 7 x 8 = ?, students can decompose the 7 into a 5 and 2, and reach the answer by multiplying 5 x 8 = 40 and 2 x 8 =16 and adding the two products (40 +16 = 56).



To further develop understanding of properties related to multiplication and division, students use different representations and their understanding of the relationship between multiplication and division to determine if the following types of equations are true or false.

- 0 x 7 = 7 x 0 = 0 (Zero Property of Multiplication)
- 1 x 9 = 9 x 1 = 9 (Multiplicative Identity Property of 1)
- 3 x 6 = 6 x 3 (Commutative Property)
- 8 ÷ 2 = 2 ÷ 8 (Students are only to determine that these are not equal)
- 2 x 3 x 5 = 6 x 5
- 10 x 2 < 5 x 2 x 2
- 2 x 3 x 5 = 10 x 3
- 0 x 6 > 3 x 0 x 2

3.OA.6: Understand division as an unknown-factor problem. For example, find 32/8 by finding the number that makes 32 when multiplied by 8. **Knowledge Targets:** ☐ Identify the multiplication problem related to the division problem. ***I can identify the multiplication problem related to the division problem.*** ☐ Identify the unknown factor in the related multiplication problem. ***I can identify the unknown factor in the related multiplication problem.*** **Reasoning Targets:** ☐ Use multiplication to solve division problems. ***I can use multiplication to solve division problems.*** ☐ Recognize multiplication and division as related operations and explain how they are related. ***I can show how multiplication and division are related operations and explain how they are related.***	This standard refers the Glossary on page 89, Table 2 (table also included at the end of this document for your convenience) and the various problem structures. Since multiplication and division are inverse operations, students are expected to solve problems and explain their processes of solving division problems that can also be represented as unknown factor multiplication problems. Example: A student knows that 2 x 9 = 18. How can they use that fact to determine the answer to the following question: 18 people are divided into pairs in P.E. class? How many pairs are there? Write a division equation and explain your reasoning. Multiplication and division are inverse operations and that understanding can be used to find the unknown. Fact family triangles demonstrate the inverse operations of multiplication and division by showing the two factors and how those factors relate to the product and/or quotient. Examples: 3 x 5 = 15 5 x 3 = 15 15 ÷ 3 = 5 15 ÷ 5 = 3 	

3.OA.9: Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations.

Knowledge Targets:

- ☐ Identify arithmetic patterns (such as even and odd numbers, patterns an an addition table, patterns in a multiplication table, patterns regarding multiples and sums.)

I can identify arithmetic patterns(such as even and odd numbers, patterns in an addition table, patterns in a multiplication table, patterns regarding multiples and sums)

Reasoning Targets:

- ☐ Explain rules for a pattern using properties of operations. (Properties of operations, glossary page 90 Common Core State Standards.)

I can explain rules for a pattern using properties of operations.

- ☐ Explain relationships between the numbers in a pattern.

I can explain relationships between numbers in a pattern.

This standard calls for students to examine arithmetic patterns involving both addition and multiplication.
Arithmetic patterns are patterns that change by the same rate, such as adding the same number. For example, the series 2, 4, 6, 8, 10 is an arithmetic pattern that increases by 2 between each term.

This standards also mentions identifying patterns related to the properties of operations.

Examples:

- Even numbers are always divisible by 2. Even numbers can always be decomposed into 2 equal addends (14 = 7 + 7).
- Multiples of even numbers (2, 4, 6, and 8) are always even numbers.
- On a multiplication chart, the products in each row and column increase by the same amount (skip counting).
- On an addition chart, the sums in each row and column increase by the same amount.

What do you notice about the numbers highlighted in pink in the multiplication table?
Explain a pattern using properties of operations.
When (commutative property) one changes the order of the factors they will still gets the same product, example 6 x 5 = 30 and 5 x 6 = 30.

x	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

Teacher: What pattern do you notice when 2, 4, 6, 8, or 10 are multiplied by any number (even or odd)?
Student: The product will always be an even number.
Teacher: Why?

x	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

What patterns do you notice in this addition table? Explain why the pattern works this way?

x	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

	Students need ample opportunities to observe and identify important numerical patterns related to operations. They should build on their previous experiences with properties related to addition and subtraction. Students investigate addition and multiplication tables in search of patterns and explain why these patterns make sense mathematically. Example: • Any sum of two even numbers is even. • Any sum of two odd numbers is even. • Any sum of an even number and an odd number is odd. • The multiples of 4, 6, 8, and 10 are all even because they can all be decomposed into two equal groups. • The doubles (2 addends the same) in an addition table fall on a diagonal while the doubles (multiples of 2) in a multiplication table fall on horizontal and vertical lines. • The multiples of any number fall on a horizontal and a vertical line due to the commutative property. • All the multiples of 5 end in a 0 or 5 while all the multiples of 10 end with 0. Every other multiple of 5 is a multiple of 10. Students also investigate a hundreds chart in search of addition and subtraction patterns. They record and organize all the different possible sums of a number and explain why the pattern makes sense.	

addend	addend	sum
0	20	20
1	19	20
2	18	20
3	17	20
4	16	20
☐	☐	☐
☐	☐	☐
☐	☐	☐
20	0	20

Geometry

3.G.1 Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four	In second grade, students identify and draw triangles, quadrilaterals, pentagons, and hexagons. Third graders build on this experience and further investigate quadrilaterals (technology may be used during this exploration). Students recognize shapes that are and are not	

sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.

Knowledge Targets:

☐ Identify and define rhombuses, rectangles, and squares as examples of quadrilaterals based on their attributes.

I can identify and define rhombuses, rectangles, and squares as examples of quadrilaterals based on their attributes.

Reasoning Targets:

☐ Describe, analyze, and compare properties of two-dimensional shapes.

I can describe, analyze and compare properties of 2D shapes.

☐ Compare and classify shapes by attributes, sides and angles.

I can compare and classify shapes by attributes, sides and angles.

☐ Group shapes with shared attributes to define a larger category (e.g., quadrilaterals).

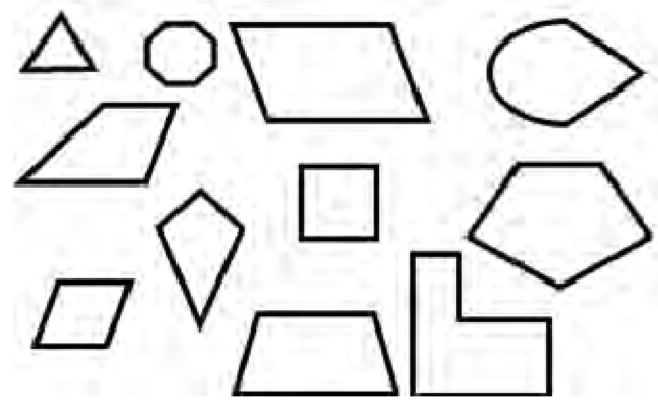
I can group shapes with shared attributes to define a larger category (eg: quadrilaterals).

Product Targets:

☐ Draw examples of quadrilaterals that do and do not belong to any of the subcategories.

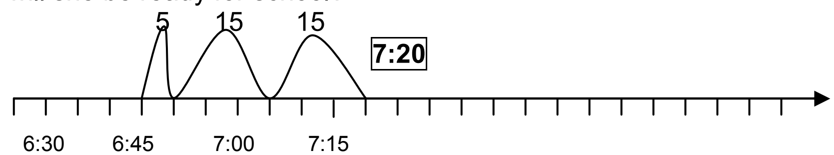
I can draw examples of quadrilaterals that do and do

quadrilaterals by examining the properties of the geometric figures. They conceptualize that a quadrilateral must be a closed figure with four straight sides and begin to notice characteristics of the angles and the relationship between opposite sides. Students should be encouraged to provide details and use proper vocabulary when describing the properties of quadrilaterals. They sort geometric figures (see examples below) and identify squares, rectangles, and rhombuses as quadrilaterals.

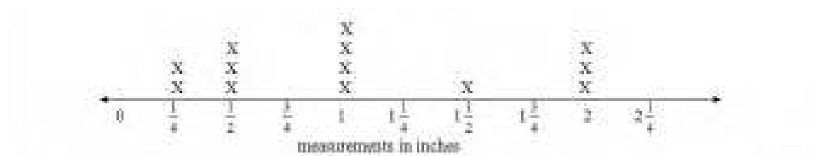


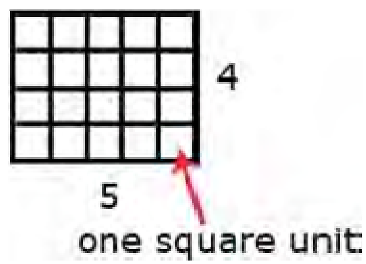
Students should classify shapes by attributes and drawing shapes that fit specific categories.
For example, parallelograms include: squares, rectangles, rhombi, or other shapes that have two pairs of parallel sides. Also, the broad category quadrilaterals include all types of parallelograms, trapezoids and other four-sided figures.

Example:
Draw a picture of a quadrilateral. Draw a picture of a rhombus.
How are they alike? How are they different?
Is a quadrilateral a rhombus? Is a rhombus a quadrilateral? Justify your thinking.

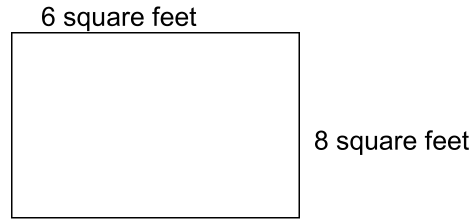
not belong to any of the subcategories.		
Measurement		
3.MD.1: Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram. **Knowledge Targets:** ☐ Recognize minute marks on analog clock face and minute position on digital clock face. ***I can recognize minute marks on analog clock face and minute position on digital clock face.*** ☐ Know how to write time to the minute. ***I can write time to the minute.*** ☐ Tell time to the minute. ***I can tell time to the minute.*** **Reasoning Targets:** ☐ Compare an analog clock face with a number line diagram. ***I can compare an analog clock face with a number line diagram.*** ☐ Use a number line diagram to add and subtract time intervals in minutes. ***I can use a number line diagram to add and subtract time intervals in minutes.*** ☐ Solve word problems involving addition and subtraction of time intervals in minutes. ***I can solve word problems involving addition and subtraction of time intervals in minutes.***	This standard calls for students to solve elapsed time, including word problems. Students could use clock models or number lines to solve. On the number line, students should be given the opportunities to determine the intervals and size of jumps on their number line. Students could use pre-determined number lines (intervals every 5 or 15 minutes) or open number lines (intervals determined by students). Example: Tonya wakes up at 6:45 a.m. It takes her 5 minutes to shower, 15 minutes to get dressed, and 15 minutes to eat breakfast. What time will she be ready for school?  	February

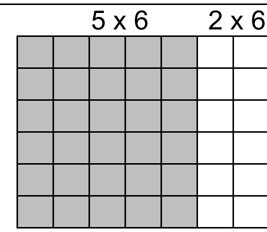
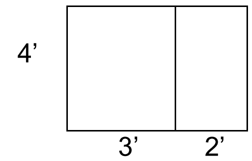
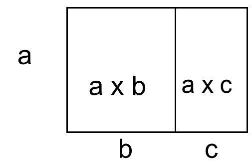
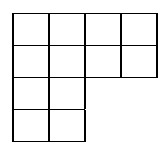
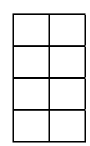
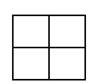
Performance Skill Targets: ☐ *Tell time to the minute.* ***I can tell time to the minute.***		
3.MD.2: Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). Add, subtract, multiply or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g. by using drawings (such as a beaker with a measurement scale) to represent the problem. **Knowledge Targets:** ☐ Explain how to measure liquid volume in liters. ***I can explain how to measure liquid volume in liters.*** ☐ Explain how to measure mass in grams and kilograms. ***I can explain how to measure mass in grams and kilograms.*** ☐ Add, subtract, multiply and divide units of liters, grams, and kilograms. ***I can add, subtract, multiply and divide units of liters, grams, and kilograms.*** ☐ Know various strategies to represent a word problem involving liquid volume or mass. ***I can identify various strategies to represent a word problem involving liquid volume or mass.*** **Reasoning Targets:** ☐ Solve one-step word problems involving masses given in the same units. ***I can solve one step word problem involving masses given in the same units.*** ☐ Solve one-step word problems involving liquid	This standard asks for students to reason about the units of mass and volume. Students need multiple opportunities weighing classroom objects and filling containers to help them develop a basic understanding of the size and weight of a liter, a gram, and a kilogram. Milliliters may also be used to show amounts that are less than a liter. Word problems should only be one-step and include the same units. Example: Students identify 5 things that weigh about one gram. They record their findings with words and pictures. (Students can repeat this for 5 grams and 10 grams.) This activity helps develop gram benchmarks. One large paperclip weighs about one gram. A box of large paperclips (100 clips) weighs about 100 grams so 10 boxes would weigh one kilogram. Example: A paper clip weighs about a) a gram, b) 10 grams, c) 100 grams? Foundational understandings to help with measure concepts: Understand that larger units can be subdivided into equivalent units (partition). Understand that the same unit can be repeated to determine the measure (iteration). Understand the relationship between the size of a unit and the number of units needed (compensatory principle).	

volume given in the same units. ***I can solve one step word problems involving liquid volume given in the same units.*** **Performance Targets:** ☐ Measure liquid volumes using standard units of liters. ***I can find liquid volume using appropriate measurement.*** ☐ Measure mass of objects using standard units of grams (g), and kilograms (kg). ***I can measure mass of objects using standard units of grams (g) and kilograms (kg).***		
3.MD.4: Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by marking a line plot, where the horizontal scale is marked off in appropriate units- whole numbers, halves, and quarters. **Knowledge Targets:** ☐ Define horizontal axis ***I can define horizontal axis.*** ☐ Identify each plot on the line as data or a number of d ***I can identify each plot on the line as data or a numb*** **Reasoning Targets:** ☐ Analyze data from a line plot. ***I can analyze data from a line plot.*** ☐ Determine appropriate unit of measurement. ***I can determine appropriate unit of measurement.*** ☐ Determine appropriate scale for line plot. ***I can determine appropriate scale for line plot.*** **Performance Skills Targets:**	Students in second grade measured length in whole units using both metric and U.S. customary systems. It's important to review with students how to read and use a standard ruler including details about halves and quarter marks on the ruler. Students should connect their understanding of fractions to measuring to one-half and one quarter inch. Third graders need many opportunities measuring the length of various objects in their environment. This standard provides a context for students to work with fractions by measuring objects to a quarter of an inch. Example: Measure objects in your desk to the nearest . or . of an inch, display data collected on a line plot. How many objects measured .? .? etc… Objects in my Desk 	

☐ Generate measurement data by measuring lengths using rules marked with halves and fourths of an inch. ***I can create measurement data by measuring lengths using rulers marked with halves and fourths of an inch.*** **Product Targets:** ☐ Create a line plot where the horizontal scale is marked off in appropriate units- whole numbers, halves, or quarters. ***I can create a line plot, where the horizontal scale is marked off in appropriate units – whole numbers, halves, or quarters.***		
3.MD.5: Recognize areas as an attribute of plane figures and understand concepts of area measurement. **a. A square with side length l unit, called "a unit square" is said to have "one square unit" of area, and can be used to measure area.** **b. A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.** **Knowledge Targets:** ☐ Define "unit square". ***I can define "unit square".*** ☐ Define area. ***I can define area.*** **Reasoning Targets:** ☐ Relate the number (n) of unit squares to the area of a plane figure. ***I can relate the number (n) of unit squares to the area of a plane figure.*** **Performance Targets:**	These standards call for students to explore the concept of covering a region with "unit squares," which could include square tiles or shading on grid or graph paper. 	

☐ Cover the area of a plane figure with unit squares without gaps or overlaps. **I can cover the area of a plane figure with unit squares without gaps or overlaps.**		
3.MD.6: Measure areas by counting unit squares (square, cm, square, m, square in, square ft, and improvised units). **Knowledge Targets:** ☐ Measure areas by counting unit squares. ***I can measure areas by counting unit squares.*** ☐ Use unit squares of cm, m, in, ft, and other sizes of unit squares to measure area. ***I can use unit squares of cm, m, in, ft, and other sizes of unit squares to measure area.***	Students should be counting the square units to find the area could be done in metric, customary, or non-standard square units. Using different sized graph paper, students can explore the areas measured in square centimeters and square inches.	
3.MD.7: Relate areas to the operations of multiplication and addition. **a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.** **Knowledge Targets:** ☐ Find the area of a rectangle by tiling it in unit squares. ***I can find the area of a rectangle by tiling it in unit squares.*** ☐ Find the side lengths of a rectangle in units. ***I can find the side lengths of a rectangle in units.***	Students should tile rectangle then multiply the side lengths to show it is the same. To find the area one could count The squares or multiply 3 x 4 = 12 <table><tr><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>5</td><td>6</td><td>7</td><td>8</td></tr><tr><td>9</td><td>10</td><td>11</td><td>12</td></tr></table>	

Reasoning Targets: ☐ Compare the area found by tiling a rectangle to the area found by multiplying the side lengths. ***I can compare the area found by tiling a rectangle to the area found by multiplying the side lengths.***		
3.MD.7: Relate areas to the operations of multiplication and addition. **b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems; and represent whole-number products as rectangular areas in mathematical reasoning.** **Knowledge Targets:** ☐ Multiply side lengths to find area of rectangles. ***I can multiply side lengths to find areas of rectangles.*** **Reasoning Targets:** ☐ Solve real world and mathematical area problems by multiplying side lengths of rectangles. ***I can solve real world and mathematical area problems by multiplying side lengths of rectangles.*** ☐ Use rectangular arrays to represent whole number products in multiplication problems. ***I can use rectangular arrays to represent whole number products in multiplication problems.***	Students should solve real world and mathematical problems. Example: Drew wants to tile the bathroom floor using 1 foot tiles. How many square foot tiles will he need? 	
3.MD.7: Relate areas to the operations of multiplication and addition. **c. Use tiling to show in a concrete case that**	This standard extends students' work with the distributive property. For example, in the picture below the area of a 7 x 6 figure can be determined by finding the area of a 5 x 6 and 2 x 6 and adding the two sums.	

the area of a rectangle with whole-number side lengths a and b + c is the sum of a x b and a x c. Use area models to represent the distributive property in mathematical reasoning. **Knowledge Targets:** ☐ Multiply using an area model (array). ***I can multiply using an area model (array).*** **Reasoning Targets:** ☐ Relate area of a rectangle to multiplication and addition by modeling the distributive property: Area of a rectangle 3 x (5+2) = 3 x 5 + 3 x 2 **I can explain the relationship of area of a rectangular to multiplication and addition by modeling the distributive property:** ***Area of a rectangle 3 x (5 + 2) = 3 x 5 + 3 x 2***	   4 x 3 + 4 x 2 = 20 4 (3 +2) = 20 4 x 5 = 20	
3.MD.7: Relate areas to the operations of multiplication and addition. **d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.** **Knowledge Targets:** ☐ Find areas of rectangles. ***I can find areas of rectangles.*** ☐ Add areas of rectangles. ***I can add areas of rectangles.***	This standards uses the word rectilinear. A rectilinear figure is a polygon that has all right angles.  How could this figure be decomposed to help find the area?  This portion of the decomposed figure is 4 x 2  This portion of the decomposed figure Is 2 x 2	

☐ Recognize that areas of each rectangle in a rectilinear (straight line) figure can be added together to find the area of the figure.
I can identify the areas of each rectangle in a rectilinear (straight line) figure can be added together to find the area of a figure.

Reasoning Targets:

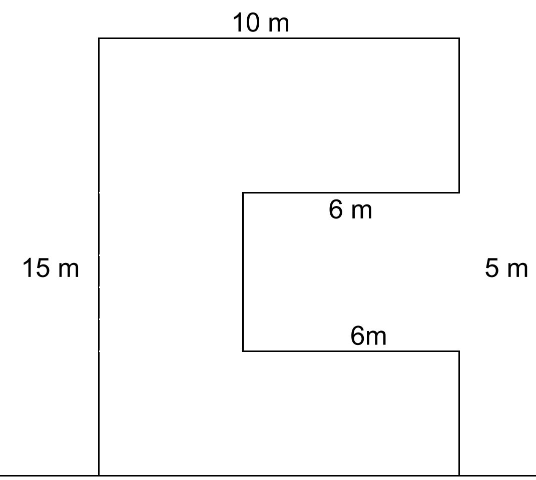
☐ Use the technique of decomposing rectilinear figures to find the area of each rectangle to solve real world problems.
I can decompose rectilinear figures to find the area of each rectangle to solve real world problems. (That means I can break apart figures)

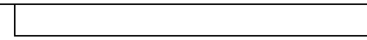
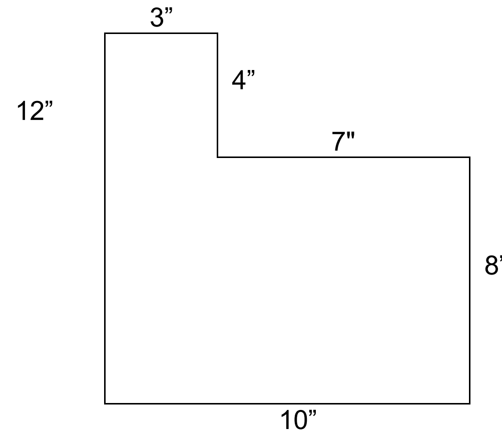
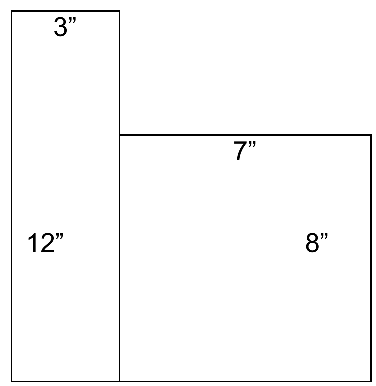
Performance Targets:

☐ Decompose rectilinear figures into non-overlapping rectangles.
***I can break apart r*ectilinear figures into separate nonoverlapping rectangles.**

4 x 2 = 8 and 2 x 2 = 4
So 8 + 4 = 12
Therefore the total area of this figure is 12 square units.

Example: A storage shed is pictured below. What is the total area? How could the figure be decomposed to help find the area?



	 Example: Students can decompose a rectilinear figure into different rectangles. They find the area of the figure by adding the areas of each of the rectangles together.   Area is 12 x 3 + 8 x 7 = 92 sq inches	
3.MD.8: Solve real world and mathematical problems involving perimeters of polygons,	Students develop an understanding of the concept of perimeter by walking around the perimeter of a room, using rubber bands to represent the perimeter of a plane figure on a geoboard, or tracing	

including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

Knowledge Targets:

☐ Define a polygon.

I can define perimeter.

☐ Define perimeter.

I can define a polygon.

Reasoning Targets:

☐ Find the perimeter when given the length of sides.

I can find the perimeter when given the length of the sides.

☐ Find the perimeter when there is an unknown side length.

I can find the perimeter when there is an unknown side length.

Product Targets:

☐ Exhibit (design, create, draw, model, etc.0 rectangles with the same perimeter and different areas.

I can create(design, create, draw, model, etc) rectangles with the same perimeter and different areas.

☐ Exhibit rectangles with the same area and different perimeters.

I can create rectangles with the same area and different perimeters.

around a shape on an interactive whiteboard. They find the perimeter of objects; use addition to find perimeters; and recognize the patterns that exist when finding the sum of the lengths and widths of rectangles.

Students use geoboards, tiles, and graph paper to find all the possible rectangles that have a given perimeter (e.g., find the rectangles with a perimeter of 14 cm.) They record all the possibilities using dot or graph paper, compile the possibilities into an organized list or a table, and determine whether they have all the possible rectangles.

Given a perimeter and a length or width, students use objects or pictures to find the missing length or width. They justify and communicate their solutions using words, diagrams, pictures, numbers, and an interactive whiteboard.

Students use geoboards, tiles, graph paper, or technology to find all the possible rectangles with a given area (e.g. find the rectangles that have an area of 12 square units.) They record all the possibilities using dot or graph paper, compile the possibilities into an organized list or a table, and determine whether they have all the possible rectangles. Students then investigate the perimeter of the rectangles with an area of 12.

Area	Length	Width	Perimeter
12 sq in	1 in	12 in	26 in
12 sq in	2 in	6 in	16 in
12 sq in	3 in	4 in	14 in
12 sq in	4 in	3 in	14 in
12 sq in	6 in	2 in	16 in
12 sq in	12 in	1 in	26 in

The patterns in the chart allow the students to identify the factors of 12, connect the results to the commutative property, and discuss the differences in perimeter within the same area. This chart can also be used to investigate rectangles with the same perimeter. It is important to include squares in the investigation.

Fractions

3.NF.1: Understand a fraction *1/b* as the quantity formed by 1 part when as whole is partitioned into *b* equal parts; understand a fraction *a/b* as the quantity formed by *a* parts of size *1/b*.

Knowledge Targets:

- ☐ Recognize a unit fraction such as ¼ as the quantity formed when the whole is partitioned into 4 equal parts.

I can identify a unit fraction such as ¼ as the quantity formed when the whole is partitioned into 4 equal parts.

- ☐ Identify a fraction such as 2/3 and explain that the quantity formed is 2 equal parts of the whole portioned into 3 equal parts (1/3 and 1/3 of the whole 3/3).

I can identify a fraction such as 2/3 and explain that the quantity formed is 2 equal parts of the whole partitioned into 3 equal parts (1/3 and 1/3 of the whole 3/3).

Reasoning Targets:

- ☐ Express a fraction as the number of unit fractions.

I can read and write a fraction as the number of unit fractions.

- ☐ Use accumulated unit fractions to represent numbers equal to, less than and greater than one (1/3 and 1/3 is 2/3; 1/3, 1/3, 1/3 and 1/3 is 4/3).

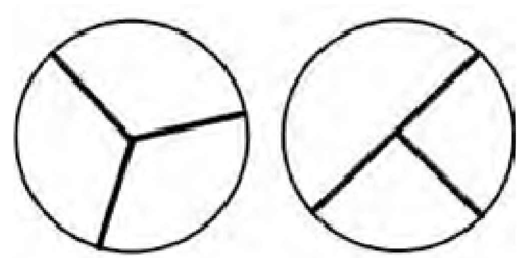
I can add unit fractions to show numbers equal to, less than and greater than one (1/3 and 1/3 is 2/3; 1/3, 1/3, 1/3, 1/3 is 4/3).

This standard refers to the sharing of a whole being partitioned or split. Fraction models in third grade include area (parts of a whole) models (circles, rectangles, squares) and number lines. Set models (parts of a group) are not introduced in Third Grade. In 3.NF.1 students should focus on the concept that a fraction is made up (composed) of many pieces of a unit fraction, which has a numerator of 1.

For example, the fraction 3/5 is composed of 3 pieces that each have a size of 1/5. Some important concepts related to developing understanding of fractions include:

- Understand fractional parts must be equal-sized.

Example Non-example

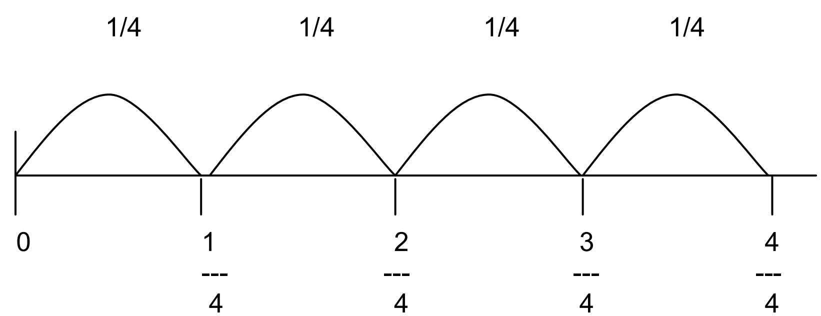


These are thirds. These are NOT thirds.

- The number of equal parts tell how many make a whole.
- As the number of equal pieces in the whole increases, the size of the fractional pieces decreases.
- The size of the fractional part is relative to the whole.
  - The number of children in one-half of a classroom is different than the number of children in one-half of a school. (the whole in each set is different therefore the half in each set will be different)
- When a whole is cut into equal parts, the denominator represents the number of equal parts.
- The numerator of a fraction is the count of the number of equal parts.
  - ¾ means that there are 3 one-fourths.
  - Students can count *one fourth, two fourths, three fourths.*

Students express fractions as fair sharing, parts of a whole, and parts

March
April

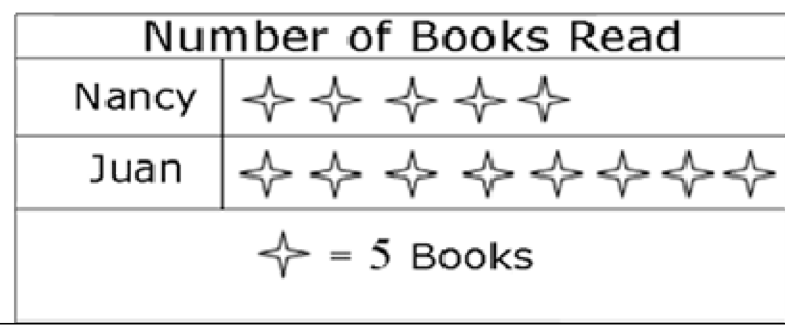
	of a set. They use various contexts (candy bars, fruit, and cakes) and a variety of models (circles, squares, rectangles, fraction bars, and number lines) to develop understanding of fractions and represent fractions. Students need many opportunities to solve word problems that require fair sharing.	
3.NF.2: Understand a fraction as a number on the number line; represent fractions on a number line diagram. **a. Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size 1/b on the number line.** **Knowledge Targets:** ☐ Define the interval from 0 to 1 on a number line as the whole. ***I can define the interval from 0 to 1 on a number line as the whole. (That means the series of fractions between 0 and 1.)*** ☐ Divide a whole on a number line into equal parts. ***I can divide a whole number on a number line into equal parts.*** ☐ Recognize that the equal parts between 0 and 1 have a fractional representation. ***I can identify that the equal parts between 0 and 1 represent fractions.*** **Reasoning Targets:** ☐ Represent each equal part on a number line with a fraction. ***I can represent each equal part on a number line with a fraction.***	The number line diagram is the first time students work with a number line for numbers that are between whole numbers (e.g., that 1/4, 1/2, 3/4 is between 0 and 1). In the number line diagram below, the space between 0 and 1 is divided (partitioned) into 4 equal regions. The distance from 0 to the first segment is 1 of the 4 segments from 0 to 1 or . (3.NF.2a). Similarly, the distance from 0 to the third segment is 3 segments that are each one-fourth long. Therefore, the distance of 3 segments from 0 is the fraction 1/4. (3.NF.2b). 	

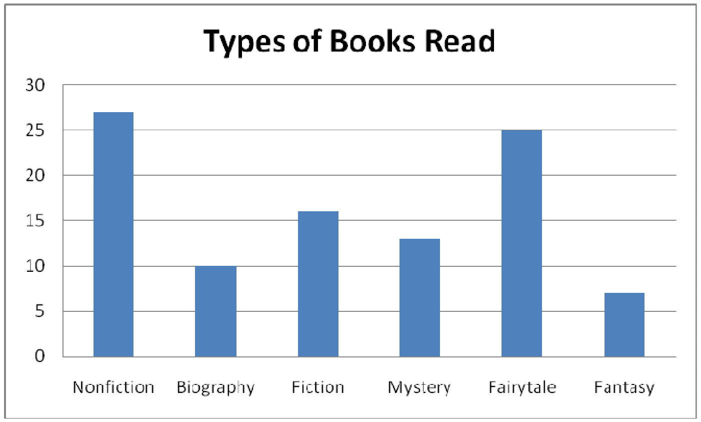
☐ Explain that the end of each equal part is represented by a fraction (1/2 the number of equal parts. ***I can explain that the end of each equal part is represented by a fraction (1/2 the number of equal parts).*** ***I can explain that the end point of each equal part represents the total number of equal parts.***		
3.NF.2: Understand a fraction as a number on the number line; represent fractions on a number line diagram. **b. Represent a fraction a/b on a number diagram by marking off a lengths 1/b from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.** **Knowledge Targets:** ☐ Define the interval from 0 to 1 on a number line as the whole. ***I can define the interval from 0 to 1 on a number line as the whole.*** ☐ Divide a whole on a number line into equal parts. ***I can divide a whole number on a number line into equal parts.*** **Reasoning Targets:** ☐ Represent each equal part on a number line with a fraction. ***I can represent each equal part on a number line with a fraction.*** ☐ Explain that the endpoint of each equal part is represents the total number of equal parts.		

|----------|----------|----------|----------| 0 ¼ 2/4 ¾ 4/4 1 ***I can explain that the end point of each equal part represents the total number of equal parts.***		
3.NF.3: Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. **a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.** **b. Recognize and generate simple equivalent fractions, (e.g., 1/2= 2/4, 4/6=2/3). Explain why the fractions are equivalent, e.g. by using a visual fraction model.** **Knowledge Targets:** ☐ Describe equivalent fractions. ***I can describe equivalent fractions.*** ☐ Recognize simple equivalent fractions. ***I can identify simple equivalent fractions.*** **Reasoning Targets:** ☐ Compare fractions by reasoning about ther size to determine equivalence. ***I can compare fractions by reasoning about their size to determine equivalence.*** ☐ Use number lines, size, visual fraction models, etc. to find equivalent fractions. ***I can use number lines, size, visual fraction models, etc. to find equivalent fractions.***	An important concept when comparing fractions is to look at the size of the parts and the number of the parts. For example, 1/8 is smaller than ½ because when 1 whole is cut into 8 pieces, the pieces are much smaller than when 1 whole is cut into 2 pieces. 3.NF.3a and 3.NF.3b These standards call for students to use visual fraction models (area models) and number lines to explore the idea of equivalent fractions. Students should only explore equivalent fractions using models, rather than using algorithms or procedures.	

3.NF.3: Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. **c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form 3=3/1; recognize that 6/1=6).** **Knowledge Targets:** ☐ Recognize whole numbers written in fractional parts on a number line. ***I can identify whole numbers written in fractional parts on a number line.*** ☐ Recognize the difference in a whole number and a fraction. ***I can identify the difference in a whole number and a fraction.*** **Reasoning Targets:** ☐ Explain how a fraction is equivalent to a whole number. ***I can explain how a fraction is equivalent to a whole number.***	This standard includes writing whole numbers as fractions. The concept relates to fractions as division problems, where the fraction 3/1 is 3 wholes divided into one group. This standard is the building block for later work where students divide a set of objects into a specific number of groups. Students must understand the meaning of a/1. Example: If 6 brownies are shared between 2 people, how many brownies would each person get?	
3.NF.3: Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. **d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when**	This standard involves comparing fractions with or without visual fraction models including number lines. Experiences should encourage students to reason about the size of pieces, the fact that 1/3 of a cake is larger than 1/4 of the same cake. Since the same cake (the whole) is split into equal pieces, thirds are larger than fourths. In this standard, students should also reason that comparisons are only valid if the wholes are identical. For example, 1/2 of a large pizza is a different amount than 1/2 of a small pizza. Students should be	

the two fractions refer to the same whole. Record the results of comparisons with the symbols >, < , or =, and justify the conclusions, e.g. by using visual fraction model. **Knowledge Targets:** ☐ Explain what the numerator in a fraction represents and its location. ***I can explain what the numerator in a fraction represents and its location.*** ☐ Explain what the denominator in a fraction represents and its location. ***I can explain what the denominator in a fraction represents and its location.*** ☐ Recognize whether fractions refer to the same whole. ***I can tell whether fractions refer to the same whole.*** **Reasoning Targets:** ☐ Determine if comparisons of fractions can be made (if they refer to the same whole). ***I can determine if comparisons of fractions can be made (if they refer to the same whole).*** ☐ Compare two fractions with the same numerator by reasoning about their size. ***I can compare two fractions with the same numerator by reasoning about their size.*** ☐ Compare two fractions with the same denominator by reasoning about their size. ***I can compare two fractions with the same denominator by reasoning about their size.*** ☐ Record the results of comparisons using symbols <, =, or >.	given opportunities to discuss and reason about which 1/2 is larger.	

I can record the results of comparisons using symbols (<,=,>). ☐ Justify conclusions about the equivalence of fractions. ***I can justify conclusions about the equivalence of fractions.***		
Maintenance and Review		
3.MD.3 Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one and two step "how many more" and "how many less" problems using information presented in scaled bar graphs. For example: draw a bar graph in which each square in the bar graph might represent 5 pets. **Knowledge Targets:** ☐ Explain the scale of a graph with a scale greater than one. ***I can explain the scale of a graph with a scale greater than one.*** ☐ Identify the scale of a graph with a scale greater than one. ***I can identify the scale of a graph with a scale greater than one.*** **Reasoning Targets:** ☐ Analyze a graph with a scale greater than one. ***I can analyze a graph with a scale greater than one.*** ☐ Choose a proper scale for a bar graph or picture graph. ***I can choose a proper scale for a bar or picture***	Students should have opportunities reading and solving problems using scaled graphs before being asked to draw one. The following graphs all use five as the scale interval, but students should experience different intervals to further develop their understanding of scale graphs and number facts. While exploring data concepts, students should Pose a question, Collect data, Analyze data, and Interpret data (PCAI). Students should be graphing data that is relevant to their lives. Example: Pose a question: Student should come up with a question. What is the typical genre read in our class? Collect and organize data: student survey Pictographs: Scaled pictographs include symbols that represent multiple units. Below is an example of a pictograph with symbols that represent multiple units. Graphs should include a title, categories, category label, key, and data. How many more books did Juan read than Nancy? 	May

graph. ☐ Interpret a bar/picture graph to solve one – or two-step problems asking "how many more" and "how many less". ***I can interpret a bar/picture graph to solve one- or two-step problems asking "how many more" and "how many less."*** **Product Targets:** ☐ Create a scaled picture graph to show data. ***I can create a scaled picture graph to show data.*** ☐ Create a scaled bar graph to show data. ***I can create a scaled bar graph to show data.***	Single Bar Graphs: Students use both horizontal and vertical bar graphs. Bar graphs include a title, scale, scale label, categories, category label, and data.  Analyze and Interpret data: • How many more nonfiction books where read than fantasy books? • Did more people read biography and mystery books or fiction and fantasy books? • About how many books in all genres were read? • Using the data from the graphs, what type of book was read more often than a mystery but less often than a fairytale? • What interval was used for this scale? • What can we say about types of books read? What is a typical type of book read? • If you were to purchase a book for the class library which would be the best genre? Why?	
3.G.2 Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For example, partition a shape into four parts with equal area, and describe the area of each part as ¼ of the area of the shape.	This standard builds on students' work with fractions and area. Students are responsible for partitioning shapes into halves, thirds, fourths, sixths and eighths. Example: This figure was partitioned/divided into four equal parts. Each part is . of the total area of the figure.	

Knowledge Targets:

☐ Know that shapes can be partitioned into equal areas.

I can show that shapes can be partitioned into equal areas.

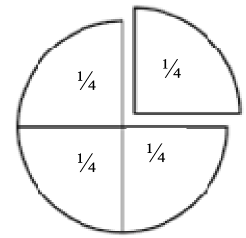
☐ Describe *the area of each part as a fractional part of the whole.*

I can describe the area of each part as a fractional part of a whole.

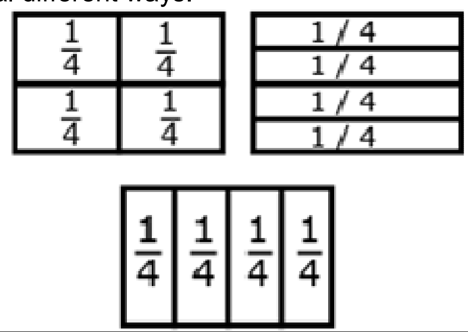
Reasoning Targets:

☐ *Relate fractions to geometry by expressing the area of part of a shape as a unit fraction of the whole. (See 3rd grade introduction).*

I can relate fractions to geometry by expressing the area of part of a shape as a unit fraction of the whole.



Given a shape, students partition it into equal parts, recognizing that these parts all have the same area. They identify the fractional name of each part and are able to partition a shape into parts with equal areas in several different ways.



Standards	***Mathematical Practices***
Students are expected to:	
3.MP.1. Make sense of problems and persevere in solving them.	In third grade, students know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Third graders may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" They listen to the strategies of others and will try different approaches. They often will use another method to check their answers.
3.MP.2. Reason abstractly and quantitatively.	Third graders should recognize that a number represents a specific quantity. They connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities.
3.MP.3. Construct viable arguments and critique the reasoning of others.	In third grade, students may construct arguments using concrete referents, such as objects, pictures, and drawings. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?" They explain their thinking to others and respond to others' thinking.
3.MP.4. Model with mathematics.	Students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Third graders should evaluate their results in the context of the situation and reflect on whether the results make sense.
3.MP.5. Use appropriate tools strategically.	Third graders consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use graph paper to find all the possible rectangles that have a given perimeter. They compile the possibilities into an organized list or a table, and determine whether they have all the possible rectangles.
3.MP.6. Attend to precision.	As third graders develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, when figuring out the area of a rectangle they record their answers in square units.
3.MP.7. Look for and make use of structure.	In third grade, students look closely to discover a pattern or structure. For instance, students use properties of operations as strategies to multiply and divide (commutative and distributive properties).
3.MP.8. Look for and express regularity in repeated reasoning.	Students in third grade should notice repetitive actions in computation and look for more shortcut methods. For example, students may use the distributive property as a strategy for using products they know to solve products that they don't know. For example, if students are asked to find the product of 7 x 8, they might decompose 7 into 5 and 2 and then multiply 5 x 8 and 2 x 8 to arrive at 40 + 16 or 56. In addition, third graders continually evaluate their work by asking themselves, "Does this make sense?"

Math Accountable Talk

Teach students to use one of the following when discussing each other's math work.

I agree with _____________ because ________________.

I like the way ___________ used ____________ because as his/her reader, it helps me _______________.

I disagree with _____________ because __________________.

I got a different answer than _______________ because ___________________.

I can add to ___________________'s thoughts: ____________________

I got the same answer as _____________________ but my strategy was different.

I have a question for ___________________________.

I don't understand why _________________ got the answer of ________________ because ________________.

Glossary

Table 1 Common addition and subtraction situations[1]

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3

	Total Unknown	Addend Unknown	Both Addends Unknown[2]
Put together/Take apart[3]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 – 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2

	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare[4]	("How many more?" version) Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 – 2 = ?	(Version with "more") Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

[2]These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.

[3]Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

[4]For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

[1]Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

Table 2 Common multiplication and division situations[1]

	Unknown Product **3 x 6 = ?**	**Group Size Unknown** **("How many in each group?" Division)** **3 x ? = 18 and 18 ÷ 3 = ?**	**Number of Groups Unknown** **("How many groups?" Division)** **? x 6 = 18 and 18 ÷ 6 = ?**
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example.* You need 3 lengths of string, each 6 inches long. How much string will you need altogether.	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example.* You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example.* You have 18 inches of string which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays[2] **Area**[3]	There are 3 rows of apples with 6 apples in each row. How man apples are there? *Area example.* What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example.* A rectangle has an area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into 3 equal rows of 6 apples, how many rows will there be? *Area example.* A rectangle has an area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example.* A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does the blue hat cost? *Measurement example.* A rubber band is 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example.* A rubber band is 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	a x b = ?	a x ? = p and p ÷ a = ?	? x b = p and p ÷ b = ?

[2]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[3]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

[1]The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

Table 3 The properties of operation
Here *a*, *b* and *c* stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

Associative property of addition	*(a + b) + c = a + (b + c)*
Commutative property of addition	*a + b = b + a*
Additive identity property of 0	*a + 0 = 0 + a = a*
Associative property of multiplication	*(a x b) x c = a x (b x c)*
Commutative property of multiplication	*a x b = b x a*
Multiplicative identity property of 1	*a x 1 = 1 x a = a*
Distributive property of multiplication over addition	*a x (b + c) = a x b + a x c*

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